in integral blackness due to destruction of the oxide film. With further heating, an oxide film is again formed on the surface of the molten metal.

German silver specimens heated in air are characterized by higher (2-3 times) values of radiation characteristics throughout the investigated temperature range compared to heating in a helium atmosphere.

The nonmonotonic nature of the ε_T curves for the oxidized specimens may be explained by the fact that the thin oxide film on the specimen surface is destroyed in the temperature range 500-1000°C and the values of ε_T drop accordingly (curves 6 and 7). The oxide film is destroyed at a lower temperature in helium.

For the investigated milled specimens of nickel and German silver heated in air, the radiation characteristics increase with temperature due to the formation of a strong oxide film (curves 3 and 5). Here, the blackness coefficients of these specimens at $t \geqslant 1000$°C correspond to the ε_T of the oxidized specimens.

Thus, heating of nickel in air at 800-1250°C is accompanied by substantial oxidation of its surface and an increase in integral blackness. Consequently, ingots are heated most rapidly in air with furnace temperatures exceeding 1000°C. This mode is evidently optimal for heating ingots in reheat furnaces and may be recommended for practical application.

LITERATURE CITED

1. K. M. Shvarev, B. A. Baum, and P. V. Gel'd, "Integral emissive power of alloys of silicon with iron, cobalt, and nickel in the temperature range from 900 to 1750°C," Teplofiz. Vys. Temp., No. 1, 78-83 (1973).
2. A. E. Sheindlin (editor), Radiation Properties of Solids [in Russian], Énergiya, Moscow (1974).

HEATING OF PLATES WITH AN ABSORPTION COEFFICIENT DEPENDENT ON TEMPERATURE AND RADIATION FLOW

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Plate heating with a temperature-dependent absorption coefficient is investigated. It is shown that with increasing power density of an incident flux the kinetics of temperature growth undergoes pronounced qualitative changes. The threshold power density value at which plate heating is accompanied by darkening as well as the darkening time are found. Calculation results are in good agreement with experimental data.

The elements of optical systems are now often subject to the effects of laser radiation flows. Semiconductor materials have found wide use in applied optics. One property of such materials is a clearly expressed dependence of the absorption coefficient on temperature. Several studies have been devoted to the heating of semiconductor plates by optical radiation. Results of empirical studies have been published in [1], e.g., and theoretical results have been presented in [2-5]. The dependence of the absorption coefficient on temperature was approximated in these works by either an exponential curve or a polynomial. It must be noted that in [2-4] the distribution function for the heat sources, determined by the Bouguer–Lambert law, was linearized. Such an approach leads to results that are less than fully satisfactory. In particular, this formulation of the problem permits an unlimited increase

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in temperature with a flow of finite capacity. This indicates a need to formulate a nonlinear problem, allowing for a change in the thermal source distribution function through the plate thickness due to absorption. Certain parameters characterizing features of the kinetics of the heating of such a plate were determined in [5].

In the action of a flow of optical radiation with a power density in the center I and having a radial distribution function f(r) on a plate of radius R and thickness 2h absorbing radiation in accordance with the Bouguer–Lambert law with an absorption coefficient $\varkappa$, the temperature T satisfies the equation

$$k\nabla^2 T - c\gamma \frac{\partial T}{\partial t} = -If(r)\varkappa(T)\exp\{-z\varkappa(T)\}. \tag{1}$$

We will assume that on the flat surfaces of the plate z = 0 and z = 2h there is a convective exchange of heat with a heat-transfer coefficient λ and a convective heat exchange on lateral surface r = R with a heat-transfer coefficient μ. We will also assume that over the entire surface, together with the convective heat exchange, there is radiative heat exchange in accordance with the Stefan–Boltzmann law and that the initial temperature of the plate is equal to the ambient temperature T_0. Thus, the boundary and initial conditions appear as follows

$$\begin{gathered} k\frac{\partial T}{\partial z} - \lambda(T - T_0) - \varepsilon\sigma(T^4 - T_0^4)\big|_{z=0} = 0, \\ k\frac{\partial T}{\partial z} + \lambda(T - T_0) + \varepsilon\sigma(T^4 - T_0^4)\big|_{z=2h} = 0, \\ k\frac{\partial T}{\partial r} + \mu(T - T_0) + \varepsilon\sigma(T^4 - T_0^4)\big|_{r=R} = 0, \\ T\big|_{t=0} = T_0. \end{gathered} \tag{2}$$

As can be seen from Fig. 1, the temperature dependence of the absorption coefficient is approximated fairly well by the exponential function $\varkappa(T) = \varkappa_0 \exp(\nu(T - T_0))$.

In many important practical applications, the effect of the radiation flow is such that the plate is heated fairly uniformly both through its thickness and along its radius. This fact finds empirical confirmation in the heating of germanium plates by a laser beam. Moreover, the estimate of the radial temperature distribution below shows the validity of the above statement, at least for a material such as germanium. This provides a basis for introducing into our investigation the deviation of the mean plate temperature from the initial

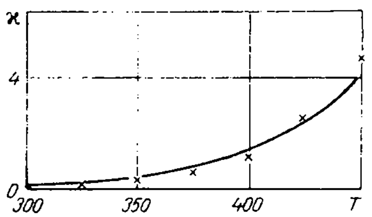

Fig. 1

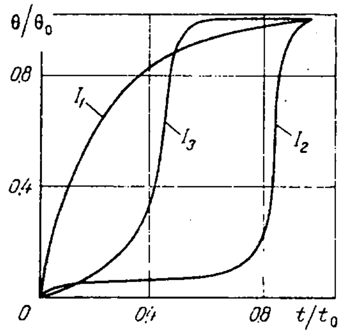

Fig. 2

Fig. 1. The exponential dependence of absorption coefficient $\varkappa$, cm^{-1}, on temperature T, °K, at $\varkappa_0 = 0.2$ cm^{-1}, $\nu = 0.02$ deg^{-1}. Experimental points, [1].

Fig. 2. Relative temperature θ/θ_0 vs relative time t/t_0, wherein t_0, time of plate heating to temperature of 0.99 θ_0.

value θ:

$$\theta = \frac{2}{R^2}\int_0^R r\frac{1}{2h}\int_0^{2h} T(r, z)\,dz\,dr - T_0. \tag{3}$$

Having averaged Eq. (1) on the assumption that $T = \theta + T_0$ and with allowance for boundary conditions (2), we obtain the equation

$$s\frac{d\theta}{dt} = Ib[1 - \exp\{-p\exp(\nu\theta)\}] - w\theta - \varepsilon\sigma v[(\theta + T_0)^4 - T_0^4]. \tag{4}$$

The following symbols are introduced at this point:

$$b = R^{-2}\int_0^R rf(r)\,dr, \quad p = 2h\varkappa_0; \quad s = hc\gamma,$$
$$v = (R + 2h)R^{-1}, \quad w = (\lambda R + 2\mu h)R^{-1}. \tag{5}$$

The solution to Eq. (4) will be found as an implicit function from the expression

$$t = \int_0^\theta \frac{s\,dx}{Ib[1 - \exp(-p\exp(\nu x))] - wx - \varepsilon\sigma v[(x + T_0)^4 - T_0^4]}. \tag{6}$$

As can be seen from the calculations, the results of which are shown in Fig. 2, there are appreciable qualitative differences in plate heating, depending on power density I. The dependence of temperature on time may either have one or two points of inflection or none at all. The values of steady-state temperature for different modes are also different. If we examine the heating of one of the plates studied experimentally in [1] ($\varkappa_0 = 0.2$ cm^{-1}; $\nu = 0.02$ deg^{-1} ; $b = 0.185$; $p = 0.14$; $v = 1.3$; $s = 3.15$ J (deg · cm^2)$^{-1}$; $w = 0.03$-0.048 W(deg · cm^2)$^{-1}$), it turns out that at low power densities — when the temperature—time dependence does not have inflection points — the plate is heated 10-20° and remains slightly absorbent, i.e., in this case most of the flow passes through the plate. At high power densities, when inflection points appear, the plate is heated hundreds of degrees and ceases to be slightly absorbent (darkening occurs). A further increase in power density leads to a shift in the first inflection point in the direction of the origin. It can be seen from the functions shown in Fig. 2 that the transition from a plate-heating mode not accompanied by darkening to one with darkening is associated with the appearance of an inflection point. With a further increase in power density, this point becomes two inflection points. Consequently, the power density I* and the corresponding steady-state temperature θ* at which the transition from a mode without darkening to a mode with darkening occurs is determined from the system of equations

$$\frac{d^2t(\theta^*, I^*)}{d\theta^2} = 0; \quad \frac{d^3t(\theta^*, I^*)}{d\theta^3} = 0. \tag{7}$$

As the calculations showed, the first inflection point of the temperature—time dependence lies in the low-temperature region, when heat exchange is mainly convective in nature. Thus, in determining threshold densities we may omit the terms describing radiant heat exchange. The problem then reduces to solution of the system of equations

$$Ibp\nu\exp\{\nu\theta - p\exp(\nu\theta)\} - w = 0;$$
$$(1 - p\exp(\nu\theta))(Ib - \exp\{-p\exp(\nu\theta)\}) - w\theta = 0. \tag{8}$$

It is not difficult to see that system (8) is equivalent to the two following systems:

$$\begin{cases} Ibp\nu \exp\{\nu\theta - p\exp(\nu\theta)\} - w = 0; \\ 1 - p\exp(\nu\theta) = 0; \end{cases} \tag{9}$$

$$\begin{cases} Ibp\nu \exp\{\nu\theta - p\exp(\nu\theta)\} - w = 0; \\ Ib(1 - \exp\{-p\exp(\nu\theta)\} - w\theta = 0. \end{cases}$$

Let us examine the first of these derived systems. Its solution has the form

$$\theta_1^* = -\frac{\ln p}{\nu}; \quad I_1^* = \frac{w}{b\nu}\exp(1). \tag{10}$$

Let us study the integrand of Eq. (6) $F(\theta, I)$. Obviously, Eq. (6) has a physical significance only in the region of the parameters and variable within which the integral is positive, i.e., we assume that the plate may be heated, but not cooled, when the effect of the radiation on the plate is unchanging. In the case being examined, for sufficiently small values of p there exists the inequality

$$F(\theta_1^*, I_1^*) = \left\{\frac{w}{s\nu}(\ln p + \exp(1) - 1)\right\}^{-1} < 0. \tag{11}$$

Consequently, the solution found does not belong to the region of permissible values of the parameters and variable. Let us examine the second system of (9). Having excluded I from the system, we obtain the following equation for determining $x = \nu\theta^*$

$$1 - (1 + px\exp(x))\exp\{-p\exp(x)\} = 0. \tag{12}$$

Having sought a solution to the equation by Newton's approximate method and having taken $x = 1$ as the first approximation, we obtain

$$x \approx \frac{p^2\exp\{2 - p\exp(1)\} + \exp\{-p\exp(1)\} - 1}{p(p\exp(1) - 1)\exp\{-p\exp(1)\}} \tag{13}$$

or with sufficiently small values of p

$$x \approx \frac{1 - p\exp(1)}{1 - 2p\exp(1)}. \tag{14}$$

Consequently, the threshold value of power density I* in this case is equal to:

$$I^* = \frac{w}{bp\nu}\exp\{p\exp(\nu\theta^*) - \nu\theta^*\} = \frac{w}{bp\nu}\exp\{p\exp(x) - x\}. \tag{15}$$

Comparing the numerically obtained solution to Eq. (12) shown in Fig. 3 with the results of calculation by Eq. (13), we may be convinced of their good agreement. The values of parameters $\varkappa_0$, ν, b, p, v, s, w are the same as in the calculation, the results of which are shown in Fig. 2. To determine the value of I_0, upon reaching which the first inflection point is shifted in the direction of the origin and heating by the flow is for the first time accompanied by an increase in the absorption coefficient, the following equation should be used

$$\left.\frac{d^2t}{d\theta^2}\right|_{\theta=0} = 0. \tag{16}$$

The sought value of power density in this case has the form

$$I_0 = \frac{w}{bp\nu}\exp(p). \tag{17}$$

It is apparent from Fig. 2 that at power densities within the interval $I^* < I < I_0$, the plate is for a time slightly absorbent. It would be useful to evaluate the duration of this period of time, since during this period the optical element remains functional. It would be appropriate to connect this time with the value of integral (6) at the upper limit, corresponding to the first point of inflection of the temperature–time dependence. Let us examine the equation

$$\frac{d^2t}{d\theta^2} = 0, \tag{18}$$

which is equivalent to the following:

$$Ibp\nu \exp\{\nu\theta - p\exp(\nu\theta)\} - w = 0. \tag{19}$$

We will again use Newton's method to find the root. Taking zero as the first approximation, we find that

$$\theta_1 \approx \frac{1}{\nu(1-p)}\left\{p + \ln\frac{w}{Ibp\nu}\right\}. \tag{20}$$

If we assume that the plate is being acted upon by a flow with the power density $I = w(2pb\nu)^{-1}$ lying within the interval $[I^*, I_0]$ being examined, then Eq. (19) takes the following form relative to $y = \nu\theta$

$$y - p\exp(y) = \ln 2. \tag{21}$$

The numerical solution to Eq. (21) shown in Fig. 3 is close to the approximate solution (20). Empirical data is presented in [1] on the darkening time of germanium specimens under the influence of laser radiation with different methods of cooling the plate edge. Figure 4 shows the results of a comparison of the theoretical and empirical data. It is apparent from this that the theoretical values are in good agreement with the experimental values.

The value of steady-state temperature is the root of the denominator of the integrand of (6). At power densities less than I*, when heating is insubstantial, the equation may be linearized and the terms associated with radiative heat exchange ignored. Consequently, the sought value θ_0 has the form

$$\theta_0^{(1)} \approx \frac{Ibp}{w - Ibp\nu}. \tag{22}$$

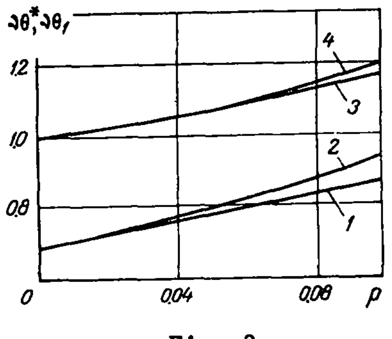

Fig. 3

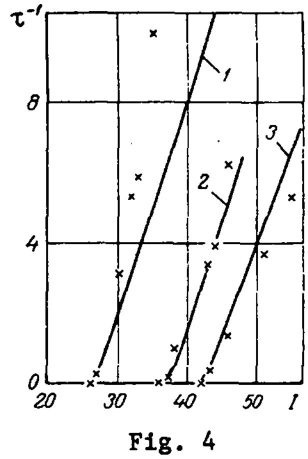

Fig. 4

Fig. 3. Dimensionless limited temperature $\nu\theta^*$ and dimensionless temperature characterizing the onset of darkening, $\theta\nu_1$, as a function of parameter p specified by initial absorptivity; 1) exact value of $\nu\theta_1$; 2) approximate value of $\nu\theta_1$; 3) exact value of $\nu\theta^*$; 4) approximate value of $\nu\theta^*$.

Fig. 4. $10^3\cdot\tau$, sec for time of plate darkening vs power density I, W/cm², for different ways of cooling the plate edge: 1) conventional air cooling; 2) cooling by kerosene at 20°C; 3) cooling by water at 20°C. Experimental points, [1].

In the case where the power density exceeds I*, darkening occurs and the plate is heated to a temperature satisfying the condition $p\exp(\nu\theta) \gg 1$. Consequently, the equation for determining $\theta_0^{(2)}$ is written thus:

$$Ib - \varepsilon\sigma v\,[(\theta_0^{(2)} + T_0)^4 - T_0^4] - w\theta_0^{(2)} = 0. \tag{23}$$

We will once more use Newton's method to determine the root. As the first approximation of $\tilde{\theta}_0^{(2)}$ we will use the value $\theta_0^{(2)}$ obtained on the assumption that the temperature is determined completely by radiative heat exchange

$$\tilde{\theta}_0^{(2)} = \left[\frac{Ib + \varepsilon\sigma v T_0^4}{\varepsilon\sigma v}\right]^{1/4} - T_0. \tag{24}$$

The approximate value of $\theta_0^{(2)}$ has the form

$$\theta_0^{(2)} \approx \frac{4\varepsilon\sigma v\omega^3(\omega - T_0)}{w + 4\varepsilon\sigma v\omega^3}; \quad \omega^4 = \frac{Ib + \varepsilon\sigma v T_0^4}{\varepsilon\sigma v}. \tag{25}$$

In a number of cases it is insufficient to know the mean temperature of the plate, and it is necessary to determine the radial temperature distribution u(r). Such a situation arises, for example, in analyzing the stress state. Values found for mean steady-state temperature (22) and (25) allow us to linearize Eq. (1), previously averaged for plate thickness. We will introduce function g(r), determined by the expression

$$u(r) = (\theta_0^{(m)} + T_0)(1 + g^{(m)}(r)). \tag{26}$$

The index m = 1, in accordance with the above, pertains to heating unaccompanied by plate darkening, while m = 2 corresponds to the case of plate darkening. Equation (26) permits us to reduce the problem of determining the radial distribution of temperature to the problem of finding unknown function $g^{(m)}(r)$ from the equation

$$\frac{1}{r}\frac{d}{dr}r\frac{dg^{(m)}}{dr} - l^{(m)}g^{(m)} = -q^{(m)} \tag{27}$$

with the boundary condition

$$\frac{dg^{(m)}}{dr} + \alpha^{(m)}g^{(m)} = \beta^{(m)}. \tag{28}$$

The following symbols are introduced at this point:

$$\begin{gathered} l^{(1)} = \frac{2\lambda - Ip\nu f(r)}{2hk}; \quad q^{(1)} = \frac{Ipf(r) - 2\lambda\theta_0^{(1)}}{2hk(\theta_0^{(1)} + T_0)}; \quad \alpha^{(1)} = \frac{\mu}{k}; \\ \beta^{(1)} = -\frac{\mu\theta_0^{(1)}}{k(\theta_0^{(1)} + T_0)}; \quad l^{(2)} = \frac{\lambda + 4\varepsilon\sigma(\theta_0^{(2)} + T_0)^3}{hk}; \\ \alpha^{(2)} = [\mu + 4\varepsilon\sigma(\theta_0^{(2)} + T_0)^3]\,k^{-1}; \\ \beta^{(2)} = -\frac{\mu\theta_0^{(2)} + \varepsilon\sigma[(\theta_0^{(2)} + T_0)^4 - T^4]}{k(\theta_0^{(2)} + T_0)}; \\ q^{(2)} = \frac{If(r) - 2\lambda\theta_0^{(2)} - 2\varepsilon\sigma[(\theta_0^{(2)} + T_0)^4 - T_0^4]}{2hk(\theta_0^{(2)} + T_0)}. \end{gathered} \tag{29}$$

In the case of a uniform radial distribution of power density (b = 0.5), the problem is further simplified and its solution has the form

$$g^{(m)}(r) = \frac{q^{(m)}}{l^{(m)}} + \frac{(\beta^{(m)}l^{(m)} - \alpha^{(m)}q^{(m)})\,I_0(r\sqrt{l^{(m)}})}{l^{(m)}\{\sqrt{l^{(m)}}\,I_1(R\sqrt{l^{(m)}}) + \alpha^{(m)}I_0(R\sqrt{l^{(m)}})\}}. \tag{30}$$

The solution found here allows us to refine the value of temperature given by Eqs. (22) and (25). If we take $I = I_0$, then for $w = 0.048$ W(deg · cm)$^{-1}$; $g(0) \approx -0.04$; $g(R) \approx -0.14$ and for $w = 0.042$ W(deg · cm)$^{-1}$; $g(0) \approx -0.05$; $g(R) \approx -0.13$.

Thus, with a flow of optical radiation with a power density less than I^*, the plate does not undergo appreciable heating and remains functional for as long as desired. If the plate is acted upon by a flow with a power density $I^* < I < I_0$, for a short time at first it is slightly absorbent. Here, the period of time over which the optical properties of the plate do not change is determined by the integral of (6) with the upper integration limit of (20). In the case where the power density is greater than I_0, heating is from the first accompanied by an intensive increase in plate temperature and darkening. In this case, the plate is functional only in the short-pulse mode.

NOTATION

R) radius of plate; h, half-thickness of plate; r, z, cylindrical coordinates; t, time; T, temperature distribution; θ, deviation of mean temperature from initial temperature; $\theta_0^{(1)}$, $\theta_0^{(2)}$, steady-state values of θ at low and high power densities of acting flux; θ_1, temperature, corresponding to beginning of sharp increase in absorption coefficient; θ^*, temperature corresponding to threshold power density; u, radial distribution of temperature; g, function characterizing radial distribution of temperature; T_0, ambient temperature; c, γ, k, specific heat, density, and thermal conductivity of plate material; λ, μ, heat-transfer coefficients on the flat and cylindrical surfaces of the plate; σ, Stefan–Boltzmann constant; ε, blackness coefficient; ϰ, absorption coefficient; $\varkappa_0$, absorption coefficient corresponding to initial temperature; ν, parameter characterizing the temperature dependence of the absorption coefficient; I, power density in center of incident flux; f, power-density distribution with respect to luminescence; I^*, I_0, threshold power densities corresponding to heatings of plate with darkening and with a sharp increase in absorption from the initial moment of the effect; p, w, s, parameters characterizing the absorption capacity of the plate at the initial temperature, Newtonian heat exchange with the environment, and heat capacity; b, averaged characteristic of power-density distribution function; v, parameter characterizing the geometric properties of the plate; x, y, notation for dimensionless temperature $\nu\theta$; τ, time of plate darkening; t_0, time of heating of the plate to $0.99\theta_0$; F, subintegral function of the solution of the equation; α, β, l, parameters characterizing heat exchange of the plate with the environment; q, parameter characterizing heat liberation in the plate.

LITERATURE CITED

1. P. A. Young, "Thermal runway in germanium laser windows," Appl. Opt., 10, No. 3, 638-643 (1971).
2. R. L. Johnson, "Thermal runway in semiconductor laser windows," Appl. Opt., 11, No. 12, 2926-2932 (1972).
3. V. L. Volod'kina and V. L. Komolov, "Kinetics of thermal breakdown of semiconductors under the influence of light," Zh. Tekh. Fiz., 45, No. 1, 134-137 (1975).
4. Khien Min' Nguen, "Toward a transient theory of the thermal breakdown of semiconductors under the influence of laser radiation," Zh. Tekh. Fiz., 45, No. 10, 2105-2108 (1975).
5. Vl. N. Smirnov, "Kinetics of semiconductor plate heating by optical radiation," Pis'ma Zh. Tekh. Fiz., 3, No. 8, 352-357 (1977).